

Titles and abstracts

Journées Complexes Lorraines 2011

Monday, May 2

14.30-15.30: Lawrence Ein (University of Illinois, Chicago). *Asymptotic syzygies of algebraic varieties*

Abstract. This is joint work with Rob Lazarsfeld. We study the asymptotic behavior of the syzygies of smooth projective varieties as the positivity of embedding line bundle grows. We prove a non-vanishing theorem for the Koszul cohomologies for such an embedding.

15.45-16.45: Sébastien Boucksom (Université Paris 6). *Ricci iteration and Kähler-Ricci on singular Fano varieties*

Abstract. I will discuss a recent joint work with Berman, Eyssidieux, Guedj and Zeriahi. Using the variational approach we developed earlier, we prove in particular the convergence of the Kähler-Ricci flow and of the Ricci iteration scheme to the Kähler-Einstein metric on Fano varieties with log-terminal singularities. In the smooth case, convergence of the Kähler-Ricci flow was previously obtained using Perelman's deep estimates.

17.15-18.15: Julien Grivaux (Université de Provence, Marseille). *The Grothendieck-Riemann-Roch theorem for abstract complex manifolds: old and new*

Abstract. The Grothendieck-Riemann-Roch theorem is one of the cornerstone of modern algebraic geometry. The corresponding results in complex geometry for various cohomology theories have been the object of numerous studies since the sixties, and many cases still remain open. In this talk, I will explain a new proof of the GRR theorem in Hodge cohomology, based on a strategy due to Kashiwara.

Tuesday, May 3

10.00-11.00: Andrei Teleman (Université de Provence, Marseille). *Towards the classifications of class VII surfaces. New approaches, new results*

Abstract. We review the program developed by the author to prove existence of curves on class VII surfaces using instanton moduli spaces, and the results obtained for $b_2 \leq 2$. We present partial results obtained for higher b_2 and the challenges which have to be overcome in order to apply successfully this program in the general case. We also discuss conjectured geometric properties of the known class VII surfaces, properties which, we believe, are essential for completing the classification.

11.30-12.30: Anand Pillay (University of Leeds). *Compact complex manifolds M with “few” subvarieties of M^n for all n*

Abstract. (Joint with Rahim Moosa.) We investigate the model theoretic notion “countable categoricity” in the context of compact complex manifolds M . When M has no positive dimensional proper subvarieties, we have a satisfactory characterization. Some questions are raised in the general case.

14.30-15.30: Joerg Winkelmann (Ruhr-Universität Bochum). *Hyperbolicity, specialness and Oka principle*

15.45-16.45: Stefan Kebekus (Albert-Ludwigs-Universität, Freiburg). *Differential Forms on Singular Spaces*

Abstract. The talk is concerned with differential forms on log canonical varieties. It is shown that any p -form defined on the smooth locus of a variety with canonical or klt singularities extends regularly to any resolution of singularities. In fact, a much more general theorem for log canonical pairs is established. The proof relies on vanishing theorems for log canonical varieties and on methods of the minimal model program. In addition, a theory of differential forms on dlt pairs is developed. It is shown that many of the fundamental theorems and techniques known for sheaves of logarithmic differentials on smooth varieties also hold in the dlt setting.

Immediate applications include the existence of a pull-back map for reflexive differentials, generalisations of Bogomolov-Sommese type vanishing results, and a positive answer to the Lipman-Zariski conjecture for klt spaces. Relations to moduli theory are emphasized.

17.15-18.15: Ngaiming Mok (HKU, Hong-Kong). *Recognizing certain rational homogeneous manifolds of Picard number 1 from their varieties of minimal rational tangents*

Abstract. Given a uniruled projective manifold X equipped with a moduli space of minimal rational curves \mathcal{K} , we associate to (X, \mathcal{K}) its varieties of minimal rational tangents (VMRTs) \mathcal{C}_x at general points $x \in X$. A fundamental question is the extent to which the manifold X is determined by its VMRTs. A special yet very interesting case of the problem is the case of characterization of a rational homogeneous manifold $S = G/P$ of Picard number 1. Given a Fano manifold of Picard number 1 (X, \mathcal{K}) equipped with a moduli space of minimal rational curves such that the VMRT $\mathcal{C}_x(X)$ at a general point $x \in X$ is biholomorphic to the VMRT $\mathcal{C}_o(S)$ of the model manifold S at a base point $o \in S$, the problem is determine whether X is biholomorphic to S . We call this the Recognition Problem for $S = G/P$.

The problem was first motivated in 2002 in the Speaker's work on a very special case of the Campana-Peternell Conjecture on compact complex manifolds with nef tangent bundle. There, assuming that X is Fano of Picard number 1 and $b_4(X) = 1$ (a condition later on removed by J.-M. Hwang in 2007), and imposing the condition that the VMRT $\mathcal{C}_x(X)$ is of dimension 1 at a general point, it was shown that X is biholomorphic to \mathbb{P}^2 , Q^3 or the 5-dimensional contact Fano homogeneous manifold $K(G_2)$. The idea was to determine VMRTs and to deduce therefrom that $X \cong S$. The Recognition Problem of $S = \mathbb{P}^n$ follows from the work Cho-Miyaoka-Shepherd-Barron. The same problem for S being Hermitian symmetric of rank ≥ 2 or of the contact type was solved by Mok in 2008, and the other cases of $S = G/P$ associated to long simple roots were later solved by Hong-Hwang. The case where S is associated to a short simple root, e.g., the case of symplectic Grassmannians (other than the Lagrangian case), remains open.

In place of just solving the Recognition Problem we propose in a certain sense to reconstruct the manifold $S = G/P$ from its VMRTs. The idea is to realize S as the base space of the Stein factorization of an iterated fibered space obtained successively from forming \mathbb{P}^1 bundles and bundles of VMRTs based on the procedure of adjunction of minimal rational curves. We will explain the approach in the simple case of the hyperquadric, where one shows that X is a hyperquadric by means of the Cartan-Fubini Principle on analytic continuation developed by Hwang-Mok.

Wednesday, May 4

10.00-11.00: Daniel Greb (Albert-Ludwigs-Universität, Freiburg). *Lagrangian fibrations on hyperkähler manifolds - On a conjecture of Beauville*

Abstract. Hyperkähler (also called irreducible holomorphic symplectic) manifolds form an important class of manifolds with trivial canonical bundle. One fundamental aspect of their structure theory is the question whether a given hyperkähler manifold admits a Lagrangian fibration. I will report on a joint project with Christian Lehn and Sönke Rollenske investigating the following question of Beauville: If a hyperkähler manifold contains a complex torus T as a Lagrangian submanifold, does it admit a (meromorphic) Lagrangian fibration with fibre T ?

11.30-12.30: Olivier Biquard (Université Paris 6). *Complex singularities and Einstein metrics*

Thursday, May 5

10.00-11.00: Julien Duval (Université Paris-Sud 11, Orsay). *Filling tori*

Abstract. We prove that a generic unknotted totally real torus of the unit sphere in the complex plane bounds a Levi-flat solid torus.

11.30-12.30: Jean-Pierre Demailly (Institut Fourier, Grenoble). *Regularity results for solutions of Monge-Ampère equations*

Abstract. Recent problems in Kähler geometry require to investigate the regularity of solutions of degenerate Monge-Ampère equations. We will present here new techniques based on the use of the Legendre-Kiselman transform (joint work with Robert Berman and Sławomir Dinew, Vincent Guedj, Pham Hoang Hiep, Sławomir Kołodziej, Ahmed Zeriahi).

14.30-15.30: Jun-Muk Hwang (KIAS Seoul). *Prolongations of the linear automorphism group of a projective variety*

Abstract. Associated to a linear Lie group $G \subset \mathbf{GL}(V)$, the k -th prolongations of G is a subspace in $\text{Hom}(S^{k+1}V, V)$ canonically determined by G , the 0-th prolongation being the Lie algebra of G . A complete classification of irreducible linear Lie groups with non-zero first prolongations is given by E. Cartan and Kobayashi-Nagano. But for non-reductive G , not much study has been done. In a joint work with Baohua Fu, we study the case when G is the linear automorphism group of a projective variety in \mathbf{PV} . We give a complete classification of irreducible non-degenerate nonsingular varieties with non-zero first prolongations, generalizing the result of Cartan and Kobayashi-Nagano.

15.45-16.45: Yum-Tong Siu (Harvard University). *Section extension from hyperbolic geometry of punctured Disk and holomorphic family of flat bundles*

Abstract. We discuss the problem of constructing pluricanonical sections from flatly twisted pluricanonical sections and prove that, for a compact complex algebraic manifold X and positive integers m and q , the subvariety of flat line bundles F on X such that the complex dimension of $H^0(X, mK_X + F)$ is at least q is regular and is a finite union of translates of abelian subvarieties by torsion elements in the abelian variety of all flat line bundles on X . The proof uses a new extension result of Ohsawa-Takegoshi type, where the curvature current is not even semi-positive but with only mild controllable negativity. We also discuss a new approach to the original theorem of Ohsawa-Takegoshi which considers the origin of the open unit 1-disk as the infinite point of the hyperbolic geometry of the punctured open unit 1-disk and reduces the original theorem of Ohsawa-Takegoshi to just a simple straightforward application of the standard method of constructing holomorphic functions by solving the $\bar{\partial}$ equation with a cut-off function and a blowup weight function.

17.15-18.15: Bruce Gilligan (University of Regina). *Kähler and locally Kähler homogeneous manifolds*

Abstract. (Joint with Karl Oeljeklaus and Christian Miebach.) We present a survey on conditions that are both necessary and sufficient for the existence of Kähler structures on some non-compact complex homogeneous manifolds and on tubular neighborhoods of some compact CR-solvmanifolds.

Friday, May 6

10.00-11.00: Akira Fujiki (Osaka University). *Some results towards moduli of anti-self-dual bihermitian structures on compact non-Kähler surfaces*

Abstract. In the previous work with M. Pontecorvo we have constructed smooth families of anti-self-dual bihermitian structures on hyperbolic and parabolic Inoue surfaces and their “anti-canonical blowing-ups”, which are universal at each of their points. In this talk we consider two general invariants for such structures, a Lee class, or equivalently a Lee bundle, and the associated Kähler cones. We then have a general result on the image of the Lee map. Also for the above families we can describe the behavior of Lee classes when the parameter tends to “infinity”. We also show that in the parabolic Inoue case the Lee class is actually identified with the moduli parameter of the surface itself, which is in contrast with the hyperbolic Inoue case.

11.00-12.30: Luca Migliorini (Universita di Bologna). *Topological properties of a class of algebraically completely integrable systems with \mathbb{C}^* action*

Abstract. I will discuss two examples, one arising from non-abelian Hodge theory, the other from Hilbert schemes of surfaces, of algebraically completely integrable systems with \mathbb{C}^* action, showing an as yet not understood behaviour. In both cases there exists another holomorphic symplectic variety whose Hodge theory reflects some topological properties of the integrable systems. If time permits, I will also discuss recent work of Chuang Diaconescu Pan giving a conjectural link of this behaviour with Gopakumar Vafa invariants.

14.30-15.30: Martin Zirnbauer (University of Cologne). *Weyl symmetry of an orbital integral transform for symmetric superspaces*

Abstract. Recent numerical and experimental studies on disordered quantum Hamiltonian systems have revealed a surprising symmetry property of the probability law for the local density of states. I will argue that the observed symmetry is related to the Weyl group invariance of an orbital integral transform for the Iwasawa decomposition of a noncompact symmetric space. More precisely, it results from an adaptation of the Iwasawa decomposition to the case of Riemannian symmetric superspaces.

16.00-17.00: Dmitri Akhiezer (Institute for Information Transmission Problems, Moscow). *70th Birthday of Alan T.Huckleberry: Laudatio*